

Mayhem Problems

To be eligible for this month's MAYHEM TAUNT, solutions must be post-marked before March 1, 2003, and attached to each solution of each problem must be a completed student information sheet.

M63. Proposed by Richard Hoshino, Dalhousie University, Halifax, Nova Scotia.

Let ABC be a right angled triangle with BC as its hypotenuse. From vertex A , construct altitude AD and internal angle bisector AE (so D and E are on side BC). We are given that $AD = 28$ and $AE = 35$. Determine the area of triangle ABC .

M64. Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario.

For real numbers x , let $f(x) = \lfloor x \lceil x \rceil \rfloor - \lceil x \lfloor x \rfloor \rceil$ (where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x and $\lceil x \rceil$ is the smallest integer greater than or equal to x).

- Show that $f(x) \geq 0$ for all $x \geq 0$, and determine when equality holds.
- What is the situation if $x < 0$?

M65. Proposed by the Mayhem Staff.

I have a number of unit cubes and I arrange them all to form a larger solid cube. I then paint some of the faces of the large cube. When the cube is disassembled it is discovered that 1000 of the cubes have no paint on them. How many faces of the big cube were painted?

M66. Proposed by Václav Konečný, Ferris State University, Big Rapids, MI, USA.

Find positive integers N in base 10 such that $N!$ in base 6 has exactly 99 trailing zeros.

M67. Proposed by J. Walter Lynch, Athens, GA, USA.

Find the interval containing r so that three consecutive terms of the geometric sequence a, ar, ar^2, \dots are the sides of a triangle.

M68. Proposed by the Mayhem Staff.

You go for a spiralling walk on the Cartesian plane. Starting at $(0, 0)$, your first five steps are to the points $(1, 0)$, $(1, 1)$, $(0, 1)$, $(-1, 1)$ and $(-1, 0)$. What point do you arrive at on your 2002th step?