Mayhem Problems

To be eligible for the April 2002 MAYHEM TAUNT, solutions must be postmarked *before* August 1, 2002, and attached to each solution of each problem must be a completed student information sheet.

M39. Proposed by the Mayhem staff.

Given x is a positive real number and

$$x = 2002 + \cfrac{1}{2002 + \cfrac{1}{2002 + \cfrac{1}{2002 + \cfrac{1}{2002 + \frac{1}{x}}}}},$$

find x.

M40. Proposed by Louis-François Préville-Ratelle, student, Cégep Régional de Lanaudière à L'Assomption, Joliette, Québec.

Suppose a and b are two divisors of the integer n, with $a < b$. Prove :

$$\left\lfloor \frac{n}{a+1} \right\rfloor + \cdots + \left\lfloor \frac{n}{b} \right\rfloor = \left\lfloor \frac{n}{\frac{n}{b}+1} \right\rfloor + \ldots + \left\lfloor \frac{n}{\frac{n}{a}} \right\rfloor .$$

Here $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

For example, if $n = 24$, $a = 3$, and $b = 6$, this says :

$$\left\lfloor \frac{24}{4} \right\rfloor + \left\lfloor \frac{24}{5} \right\rfloor + \left\lfloor \frac{24}{6} \right\rfloor = \left\lfloor \frac{24}{5} \right\rfloor + \left\lfloor \frac{24}{6} \right\rfloor + \left\lfloor \frac{24}{7} \right\rfloor + \left\lfloor \frac{24}{8} \right\rfloor ,$$

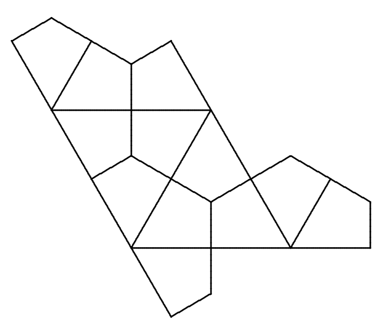
which evaluates to the identity $6 + 4 + 4 = 4 + 4 + 3 + 3$.

M41. Proposed by J. Walter Lynch, Athens, GA, USA

Find the number of orders of wins and losses that can occur in a World Series. For example if the series ends after five games there are eight possible orders : **ANNNN NANNN NNANN NNNAN NAAAA ANAAA AANAA AAANA** where **A** is for an American League win and **N** is for a National League win. Note that the series ends as soon as one team wins four games.

M42. Proposed by Izidor Hafner, Tržaška 25, Ljubljana, Slovenia.

The diagram below represents the net of a polyhedron. The faces of the solid are divided into smaller polygons. The task is to colour the polygons (or number them), so that each face of the original solid is a different colour.



M43. Proposed by the Mayhem staff.

Prove that

$$\frac{29 - 5\sqrt{29}}{58} \left(\frac{7 + \sqrt{29}}{2} \right)^{2002} + \frac{29 + 5\sqrt{29}}{58} \left(\frac{7 - \sqrt{29}}{2} \right)^{2002}$$

is an integer.

M44. Proposed by K.R.S. Sastry, Bangalore, India.

$ABCD$ is a Heron parallelogram (in which the sides, the diagonals and the area are natural numbers). The diagonals AC and BD have measures 85 and 41 respectively. Determine the measures of the sides AB and BC.