

Mayhem Problems

To be eligible for the February 2002 MAYHEM TAUNT, solutions must be postmarked *before June 1, 2002*, and attached to each solution of each problem must be a completed student information sheet.

M29. *Proposed by the Mayhem staff.*

Define the “silly product” of two numbers as the sum of the product of all the corresponding digits. So $235 \times_s 718 = 2 \times 7 + 3 \times 1 + 5 \times 8 = 57$. Find two numbers A and B so that $A \times_s B = 2002$ and $A + B$ is a minimum.

M30. *Proposed by Haralampy Steryion, Chalkis, Greece.*

Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ with the property

$$f(x + y) = f(x)e^{f(y)-1} \quad \text{for every } x, y \in \mathbb{R}.$$

M31. *Proposed by the Mayhem staff.*

Given four spheres of unit radius, each tangent to the other three, find the radii of the two spheres that are tangent to all four of the unit spheres.

M32. *Proposed by Nicolae Gustia, North York, Ontario.*

In a triangle with angles A , B and C , if $8 \cos A \cos B \cos C = 1$ then prove that $\triangle ABC$ is equilateral.

M33. *Proposed by Richard Hoshino, Dalhousie University, Halifax, Nova Scotia.*

a , b and c are three consecutive terms of a geometric sequence, where a , b and c are all integers. If $a + b + c = 7$, determine all possible values of a , b and c .